Exercise 78

(a) Show that $f(x) = x + e^x$ is one-to-one.

(b) What is the value of $f^{-1}(1)$?

(c) Use the formula from Exercise 77(a) to find $(f^{-1})'(1)$.

Solution

Part (a)

Calculate the derivative of $f(x)$.

$$\begin{aligned} f'(x) &= \frac{d}{dx}(x + e^x) \\ &= \frac{d}{dx}(x) + \frac{d}{dx}(e^x) \\ &= 1 + e^x \end{aligned}$$

Since $f'(x) > 0$ for all x, $f(x)$ is monotonically increasing. This means there exists a unique y-value for every x-value. Therefore, f is a one-to-one function.

Part (b)

In order to find $f^{-1}(1)$, solve the following equation for x.

$$1 = x + e^x$$

By inspection, $x = 0$, so

$$f^{-1}(1) = 0.$$

Part (c)

Use the formula from Exercise 77(a) to find $(f^{-1})'(1)$.

$$\begin{aligned} \frac{d}{dx}[f^{-1}(x)]\bigg|_{x=1} &= \frac{1}{f'[f^{-1}(1)]} \\ &= \frac{1}{f'(0)} \\ &= \frac{1}{1 + e^0} \\ &= \frac{1}{1 + 1} \\ &= \frac{1}{2} \end{aligned}$$